

Index

- Abadie's CQ, 141
- active constraint ($I(x)$), 95
- adjacent extreme point, 235
- affine independence, 37
- affine function, 14, 66
- affine subspace, 36
- affine transformation, 433
- algebraic characterization of adjacency, 236
- approximate line search, 300
- Armijo step, 301, 332, 431
- artificial variables, 247
- augmented Lagrangian function, 441
- augmented Lagrangian method, 441
- automatic differentiation, 322

- Banach's Theorem, 117
- barrier function, 378
- barrier problem, 379
- basic feasible solution, 229
- basic solution, 229
- basic variables, 229
- basis, 37
- BFGS method, 296
- Bland's rule, 252
- boundary, 40
- bounded set, 40
- Brouwer's Theorem, 117
- bundle method, 182

- calculus rules, 42
- canonical form, 257
- Carathéodory's Theorem, 48
- Cartesian product set, 164
- Cauchy-Bunyakovski-Schwarz inequality, 36

- central difference formula, 321
- centralized planning, 285
- classification of optimization models, 14
- closed sets, 40
- closed mapping, 175, 344
- closure, 40
- coercive function, 84
- column dropping, 329
- column generation, 9
- combinatorial optimization, 201
- complementarity, 164
- Complementary Slackness Theorem, 264
- composite function, 67, 121
- composite operator, 121
- concave function, 66
- cone, 53
 - polar, 114
 - polyhedral, 53
- cone of feasible directions, 129
- conjugate direction, 310, 319
- conjugate gradient, 313
- conjugate gradient method, 313
- constrained optimization, 14, 94-103, 323-396, 436-441
- constraint qualification (CQ), 19, 140, 141, 147, 148
- constraints, 5
- continuity, 103
- continuous function, 40
- continuous optimization, 14
- continuous relaxation, 201
- continuously differentiable function, 41

Index

- contractive operator, 117
- convergence rate, 320
 - geometric, 117, 186
 - linear, 320
 - quadratic, 321
 - superlinear, 320
- convex problem, 150
- convex analysis, 43–77, 402–405
- convex function, 65, 103, 175
- convex problem, 82, 160, 165, 181
- convex programming, 15
- convex set, 43
- coordinates, 37
- CQ, 140

- Danskin's Theorem, 176
- Dantzig-Wolfe algorithm, 174
- decision science, 12
- decision variable, 8
- degenerate basic solution, 229
- descent direction, 92, 181
- descent lemma, 436
- DFP method, 317
- Diet problem, 13
- differentiability, 179
- differentiable function, 41
- differentiable optimization, 15
- Dijkstra's algorithm, 340
- diode, 198
- direction of unboundedness, 240
- directional derivative, 40, 93, 175
- distance function, 75
- divergent series step length rule, 181, 328
- domination, 394
- dual feasible basis, 269
- dual infeasible basis, 269
- dual linear program, 256
- dual simplex algorithm, 270
- dual simplex method, 268
- duality gap, 161

- effective domain, 103, 160
- efficient frontier, 394
- eigenvalue, 39
- eigenvector, 39
- Ekeland's variational principle, 410

- electrical circuit, 197
- electrical network, 197
- eligible entering variable, 252
- eligible leaving variable, 252
- epigraph, 69, 86
- ϵ -optimal solution, 103
- equality constraint, 14
- equivalent systems, 423
- Euclidean projection, 74
- Everett's Theorem, 193
- exact penalty function, 388
- existence of optimal solution, 233
- exterior penalty method, 374–378
- extreme direction, 232
- extreme point, 49

- Farkas' Lemma, 60, 105, 109, 155, 263
- Farkas' Lemma for an inequality system, 60
- feasibility heuristic, 204
- feasible direction, 95
- feasible solution, 7, 16
- feasible-direction methods, 323
- filter, 394
- filter-SQP methods, 394
- finite termination, 305
- finitely generated cone, 63
- fixed point, 116
- Fletcher-Reeves formula, 316
- forward difference formula, 321
- Fourier elimination, 56
- Frank-Wolfe algorithm, 325
- Frank-Wolfe Theorem, 88
- Fritz John conditions, 136
- full rank, 38

- Gauss-Newton method, 297–299
- Gauss-Seidel method, 83
- Gauss-Seidel method, 121
- geometric convergence rate, 117, 186
- global minimum, 82
- global optimality conditions, 150
- global optimality conditions, 163
- global optimum, 82
 - necessary and sufficient conditions, 94, 97

- sufficient conditions, 150
- Golden section, 301
- Gordan's Theorem, 62, 113
- gradient, 41
- gradient projection algorithm, 332
- gradient related, 293
- gradient related method, 303, 304
- Gram-Schmidt procedure, 313

- hard constraint, 20
- Hessian matrix, 41
- hyperplane
 - proper supporting, 110
 - supporting, 110

- $l(x)$, 95
- identity matrix I^n , 39
- ill-conditioning, 394
- implicit function, 42, 321
- Implicit Function Theorem, 180
- indicator function (χ_S), 183, 373
- inequality constraint, 14
- infimum, 17
- infinite-dimensional optimization, 16
- integer programming, 14, 15
- integrable function, 338
- integrality property, 15
- interior, 40
- interior penalty function, 123
- interior point algorithm, 253
- interior point algorithm, 378-385
- interpolation, 301
- iso-cost line, 290
- iso-curve, 290

- Jacobi method, 121
- Jacobian, 41, 338, 387, 432

- Karmarkar's algorithm, 253
- Karush-Kuhn-Tucker (KKT) conditions, 142, 150
- kernel, 38
- Kirchhoff's laws, 198

- Lagrange function, 158
- Lagrange multiplier method, 174, 190
- Lagrange multiplier vector, 159, 196
- Lagrange multipliers, 137
- Lagrangian relaxation, 21
- Lagrangian dual function, 159
- Lagrangian dual problem, 159
- Lagrangian duality, 157-208, 417-421
- Lagrangian relaxation, 158, 159, 201
- least-squares data fitting, 289
- least-squares problems, 297-299
- level curve, 290
- level set ($\text{lev}_g(b)$), 73, 74, 85, 86, 168, 183, 303, 304, 332
- Levenberg-Marquardt, 295, 433
- LICQ, 148
- limit, 39
- limit points, 40
- line search, 299
 - approximate, 300
 - Armijo step length rule, 332
 - Armijo step length rule, 301, 431
 - Golden section, 301
 - interpolation, 301
 - Newton's method, 301
- linear programming, 14
- linear convergence rate, 320
- linear function, 42
- linear independence, 37
- linear programming, 16, 170, 211-285, 384-385, 430
- linear programming duality, 255-285, 426-430
- linear space, 36
- linear subspace, 362
- linear-fractional programming, 422
- Lipschitz continuity, 304
- local minimum, 82
- local convergence, 387
- local optimum, 82
 - necessary conditions, 91, 93, 96, 132, 136, 142, 146
 - sufficient conditions, 93
- logarithmic barrier, 379
- logical constraint, 5, 7
- lower semi-continuity, 85

Index

- Maratos effect, 392
- mathematical model, 4
- mathematical programming, 11
- matrix, 37
- matrix game, 121
- matrix inverse, 38
- matrix norm, 37
- matrix product, 38
- matrix transpose, 37
- max function, 176
- mean-value theorem, 41
- merit function, 389
- method of successive averages (MSA), 371
- MFCQ, 147
- minimax theorem, 121
- minimum, 17
- minimum distance (dist_S), 183
- Minkowski-Weyl Theorem, 64, 109
- multi-objective optimization, 16, 394

- near-optimality, 102
- negative curvature, 292
- neighbourhood, 40
- Newton's method, 294, 301, 432
- Newton-Raphson method, 121, 294
- Nobel laureates, 13
- non-basic variables, 229
- non-convex programming, 15
- non-coordinability, 192
- non-differentiable function, 307
- non-differentiable optimization, 15
- non-expansive operator, 115
- nonlinear programming, 14, 16
- nonsingular matrix, 38
- nonsingular matrix, 38
- norm, 36
- normal cone (N_X), 100, 142
- NP-hard problem, 156, 202
- null space, 38

- objective function, 4
- Ohm's law, 199
- open ball, 40
- open set, 40
- operations research, 12
- optimal BFS, 240

- optimal solution, 7
- optimal value, 7
- optimality, 12
- optimality conditions, 91-94, 96-101, 125-156, 163-165, 191-192, 241, 242, 266-267, 412-416
- optimization under uncertainty, 16
- optimize, 3
- orthogonality, 36, 164
- orthonormal basis, 37

- parametric optimization, 154
- Pareto optimal solution, 16
- Pareto set, 394
- partial pricing, 244
- pattern search methods, 322
- penalty, 21
- penalty function, 21
- penalty parameter, 374
- perturbation function ($p(u)$), 194
- Phase I, 336
- phase I problem, 247
- phase II problem, 248
- physical constraint, 5, 7
- piece-wise linear function, 307
- Polak-Ribière formula, 316
- polar cone, 114
- Polyak step, 181
- polyhedral cone, 53
- polyhedron, 50
- polytope, 49
- positive (semi)definite matrix, 39
- potential, 198
- potential difference, 198
- pre-conditioning, 316
- price-directive decomposition, 282
- primal infeasibility criterion, 269
- primal simplex method, 268
- projection, 74
- projection matrix, 362
- projection arc, 332
- projection matrix, 362
- projection operator, 74, 98, 115
- projection problem, 336
- proof by contradiction, 35

- proper separation, 105
- proper function, 18, 183
- proper supporting hyperplane, 110
- proximal point algorithm, 434
- pseudo-convex function, 409

- Q-orthogonal, 310
- quadratic programming, 172
- quadratic convergence rate, 321
- quadratic function, 42, 72, 156
- quadratic programming, 156, 336
- quasi-convex function, 410
- quasi-Newton methods, 295, 317, 390

- Rademacher's Theorem, 307
- range space, 38
- rank, 38
- rank-two update, 317
- recession cone, 88
- reduced gradient, 367
- reduced cost, 241
- redundant constraint, 408
- regular vector, 19
- relaxation, 21, 157, 158, 201, 376
- Relaxation Theorem, 157
- Representation Theorem, 328
- Representation Theorem, 53, 232
- resistor, 198
- restricted master problem, 329
- restricted simplicial decomposition, 330
- restrification, 440
- revised simplex method, 253

- saddle point, 121, 122, 163, 292
- scalar product, 36
- secant method, 296
- sensitivity analysis, 154, 194, 196
- sensitivity analysis for LP, 272-276
- separation, 105
 - proper, 105
 - strict, 105
 - strong, 105
- Separation Theorem, 105, 179
- sequential linear programming (SLP), 440

- sequential quadratic programming (SQP), 385-394
- shadow price, 263
- shortest route, 340
- simplex method, 12, 239-254, 424-425
- simplicial decomposition algorithm, 328
- slack variable, 9
- Slater CQ, 148
- SLP algorithm, 440
- soft constraint, 21, 193
- solution set, 346
- spacer step, 350
- spectral theorem, 153
- SQP algorithm, 385-396, 441
- square matrix, 38
- stalling, 253
- standard basis, 37
- stationary point, 19
- stationary point, 91, 97
- steepest descent, 291
- steepest-edge rule, 244
- stochastic programming, 16
- strict separation, 105
- strict inequality, 87
- strict local minimum, 82
- strictly convex function, 66
- strictly quasi-convex function, 301
- strong separation, 105
- strong duality, 165
- Strong Duality Theorem, 165
- Strong Duality Theorem, 172
- Strong Duality Theorem, 168-170, 262
 - convex program, 165, 168, 169
 - linear program, 170, 262
 - quadratic program, 172
- subdifferentiability, 178
- subdifferential, 175
- subgradient, 174, 308
- subgradient optimization, 182
- subgradient projection method, 181
- superlinear convergence rate, 320
- supporting hyperplane, 110
- symmetric matrix, 39

Index

- tangent cone, 129
- traffic assignment problem, 339
- traffic equilibrium, 337
- triangle inequality, 38
- trust region methods, 309
- twice differentiable function, 41

- unconstrained optimization, 14, 91-94, 289-322, 431-435
- unimodal function, 301
- unique optimum, 89
- upper level set, 168
- upper semi-continuity, 85
- user equilibrium, 338

- variable, 4
- variational inequality, 97, 120
- vector, 36
- vector-valued functions, 41
- voltage source, 198
- von Neumann's Minimax Theorem, 121

- Wardrop's principle, 338
- Weak Duality Theorem, 160, 261
 - Lagrangian relaxation, 160
 - linear program, 261
- weak Wolfe condition, 302
- weakly coercive function, 84
- Weierstrass' Theorem, 86, 105, 178
- Wolfe condition, 302

